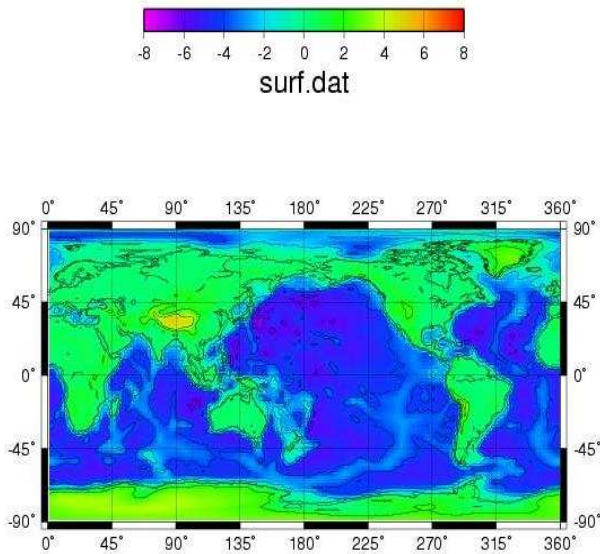# The Geoid generated by deflected surfaces

- Data visualisation and investigation of isostacy
- Surface harmonic analysis and computation of geoid effects

Report
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3.1.1 GMT plots of the topography and crustal thickness.

This GMT-plot shows the distribution of the topography in kilometres around the world. Clearly visible are the Himalaya mountainbelt and the mid oceanic ridges
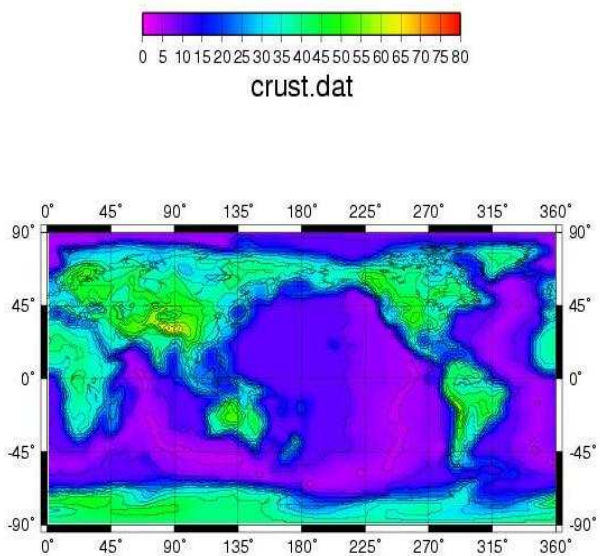
This GMT plot shows the crustal thickness in kilometres around the world. The crust is thinnest at the oceanic ridges and thickest at the mountain belts.

3.1.2 Plot of crustal thickness versus topographic height

Correlation between topographic height and crustal thickness
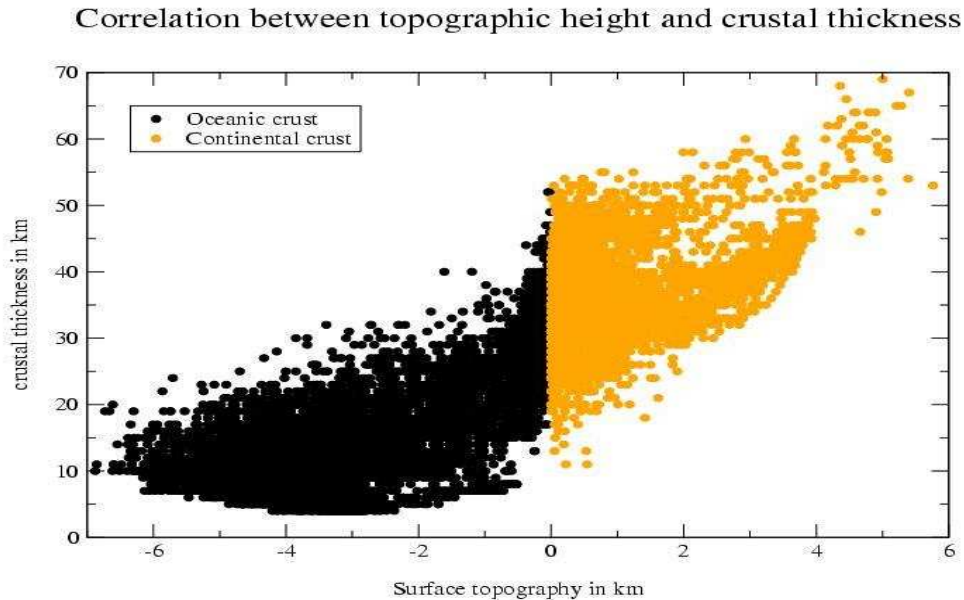
This plot shows the relationship between crustal thickness and type of crust. The lower the surface topography the thinner the crust in general. Some negative surface topography can be explained by the presence of shallow intercontinental seas, below which the crust is continental and therefore plot higher than other points. This is best visible near the zero surface topography.

The used fortran program "correlation":

```
      program correlation
      implicit none
c     declaration of variables
      integer MAXLON,MAXLAT, x
      parameter (MAXLON=360,MAXLAT=180)
      double precision topo(MAXLON,MAXLAT),thick(MAXLON,MAXLAT)
      double precision longitude(MAXLON),latitude(MAXLAT)

      integer lat,lon, nlat,nlon
      double precision minlat,dlat,minlon,dlon

c    Open and create input and output files
      open(UNIT=9, FILE='surf.dat')
      open(UNIT=12, FILE='crust.dat')
      open(UNIT=10, FILE='oceanic.dat')
      open(UNIT=11, FILE='continent.dat')

c    * skip first line file header from surf.dat
      read(9,'(a)') x
c    * read 2 lines of mesh info from surf.dat
      read(9,*) minlon,dlon,nlon
      read(9,*) minlat,dlat,nlat
c    * skip remaining 2 header lines from surf.dat
      read(9,'(a)') x
      read(9,'(a)') x
c    * skip first line file header for crust.dat
      read(12,'(a)') x
c    * read 2 lines of mesh info for crust.dat
      read(12,*) minlon,dlon,nlon
      read(12,*) minlat,dlat,nlat
c    * skip remaining 2 header lines for crust.dat
      read(12,'(a)') x
```

```fortran
      read(12,'(a)') x


c     Start the do-loops for the latitude and longitude
      do 200 lat=1,nlat
        do 100 lon=1,nlon
c     read from surf.dat and crust.dat the surface topography and crustal thickness for each coordinate.
          read(9,*) longitude(lon),latitude(lat),topo(lon,lat)
          read(12,*) longitude(lon), latitude(lat),thick(lon,lat)
c     Apply boundary conditions for separation between continental and oceanic crust
                  if (topo(lon,lat) < 0) then
                  write(10,*) topo(lon,lat), thick(lon,lat)
                  else if (topo(lon,lat) >= 0) then
                  write(11,*) topo(lon,lat), thick(lon,lat)
            end if
100     continue
200   continue

      close (9)
      close (10)
      close (11)
      close (12)

      end
```

3.1.3

The program airy used to calculate model crustal thickness and the difference between the 'modelled' isostatic crustal thickness and the observational dataset:

```fortran
      program Airy
      implicit none

c     declaration of variables
      integer MAXLON,MAXLAT
      parameter (MAXLON=360,MAXLAT=180)
      integer lat, lon, rhocr, rhom, rhow, dc
      integer nlat, nlon, minlon, dlon, minlat, dlat, x
      double precision dr(MAXLON,MAXLAT), thickness(MAXLON,MAXLAT)
      double precision topo(MAXLON,MAXLAT), ea(MAXLON,MAXLAT)
      double precision longitude(MAXLON),latitude(MAXLAT)
      double precision thickdif(MAXLON,MAXLAT), thick(MAXLON,MAXLAT)

c     values for the constants used
      rhocr=2700
      rhom=3250
      rhow=1000
      dc=35000

c     open and create input and output files
      open(UNIT=9, FILE='surf.dat')
      open(UNIT=10, FILE='thickdiff.dat')
      open(UNIT=11, FILE='crust.dat')
c     skip first line of input file
      read(9,'(a)') x
c     read 2 lines of input files
      read(9,*) minlon,dlon,nlon
      read(9,*) minlat,dlat,nlat
c     skip 2 lines in input file
      read(9,'(a)') x
      read(9,'(a)') x

c     skip first line of input file
      read(11,'(a)') x
c     read 2 lines of input files
      read(11,*) minlon,dlon,nlon
      read(11,*) minlat,dlat,nlat
```

```fortran
c     skip 2 lines in input file
      read(11,'(a)') x
      read(11,'(a)') x


c     write first five lines in style of input files to output file.
      write(10,*) 'Dif between observed & calculated crustal thickness'
      write(10,*) minlon,dlon,nlon, '  # minlon,dlon,nlon'
      write(10,*) minlat,dlat,nlat, '  # minlat,dlat,nlat'
      write(10,*) 'phi (deg),  theta (deg), thicknessdifference (km)'
      write(10,*) '--------------------------------------------------'
c     start do loop over the latitude and longitude from input file.
      do 200 lat=1,nlat
        do 100 lon=1,nlon
c     read for each latitude and longitude the associated surface topography and crustal thickness
          read(9,*) longitude(lon),latitude(lat),topo(lon,lat)
          read(11,*) longitude(lon), latitude(lat), thick(lon,lat)
c     calculate the compensating root for continental crust
          if (topo(lon,lat) >= 0) then
        dr(lon,lat) = 1000*((rhocr/(rhom-rhocr))*topo(lon,lat))
          thickness(lon,lat) = dr(lon,lat) + dc + (1000*topo(lon,lat))
c     calculate the anti-root for oceanic crust
          else if (topo(lon,lat) < 0) then
        ea(lon,lat) = ((rhocr-rhow)/(rhom-rhocr))*(1000*-topo(lon,lat))
          thickness(lon,lat) = dc - ea(lon,lat) + (1000*topo(lon,lat))
          end if
c     calculate the difference between the crustal thickness(observed) and crustal thickness(calculated)
        thickdif(lon,lat) = thick(lon,lat) - (thickness(lon,lat)/1000)
c        write data to output file
          write(10,*) longitude(lon),latitude(lat), thickdif(lon,lat)
100      continue
200    continue
      close(9)
      close(10)
      close(11)
      end
```

The mismatch values in a GMT plot
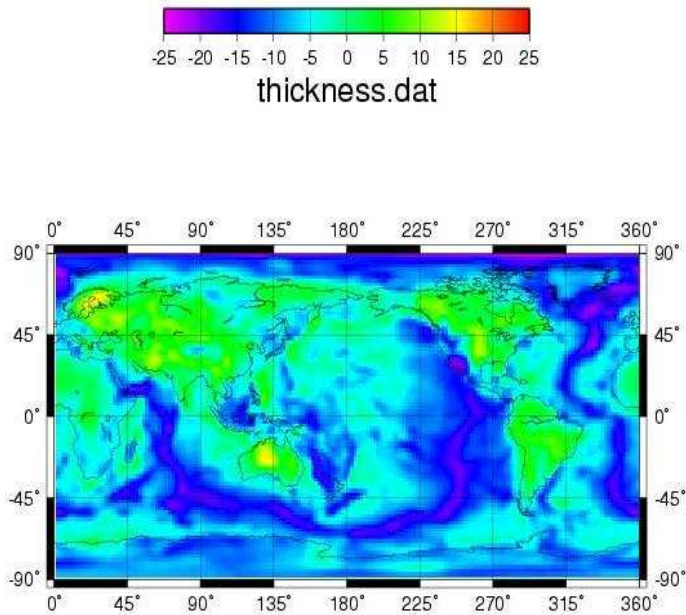
thickness.dat

The difference in crustal thickness is shown in kilometres. The old continents show a positive difference in crustal thickness such as Australia and Scandinavia. On the other hand, the oceanic ridges show a highly negative difference. The crust here is very young and not isostatically compensated yet. However, the old continents are not isostatically compensated as well, because a lot of material has been eroded away. Thus it seems that erosion happens faster than isostatic compensation.

3.2.1 A few simple low degree surface harmonic functions in GMT plots
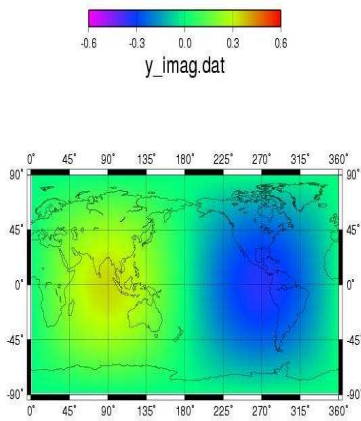
1*1 (imaginary part)   1*1 (real part)
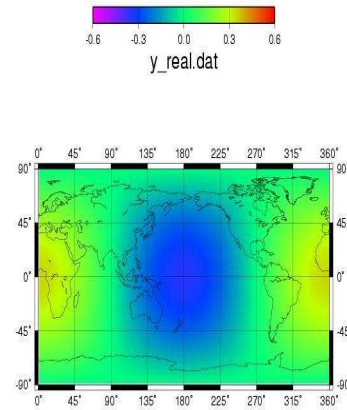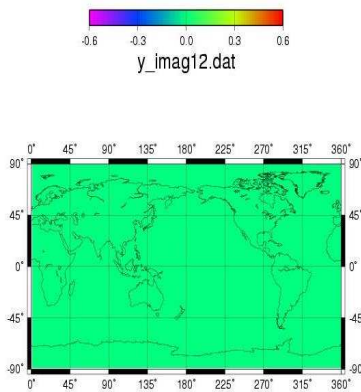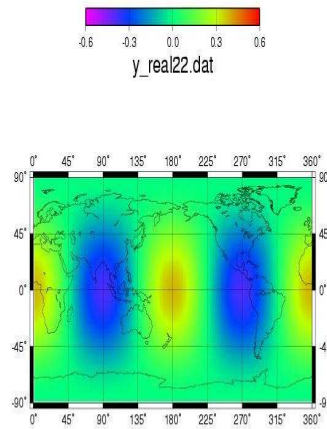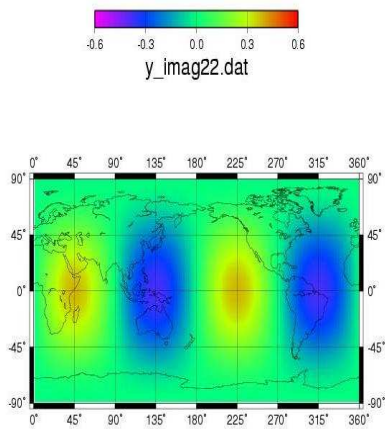

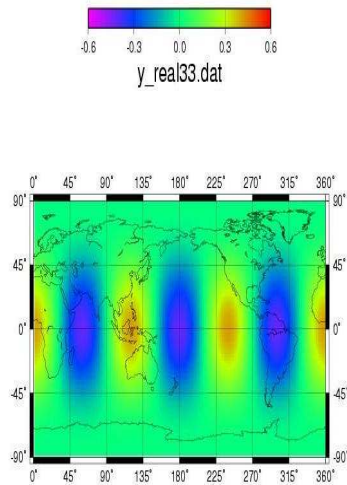
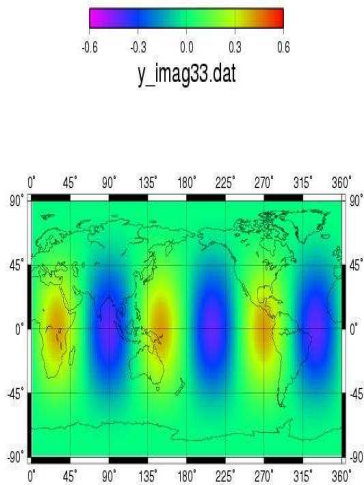1*2 (imaginary part)   1*2 (real part)
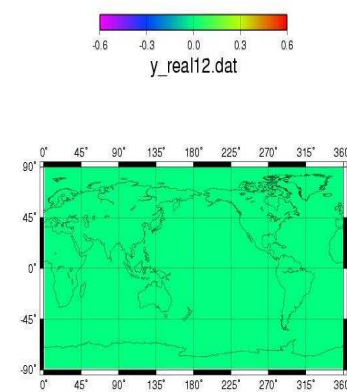
2*2 (imaginary part)   2*2 (real part)

y_imag22.dat

y_real22.dat

3*3 (imaginary part)   3*3 (real part)


y_imag33.dat

y_real33.dat

Summary

As you can see, as the surface harmonics, j and m, are increased the complexity increases as well to a greater precision. For 1*2 the result is zero for both the real and imaginary part because the order (j) is always equal or greater than the degree (m), so 1*2 is not possible but 2*1 is possible. I.e. m counts until the highest value of j each sum and cannot be greater.
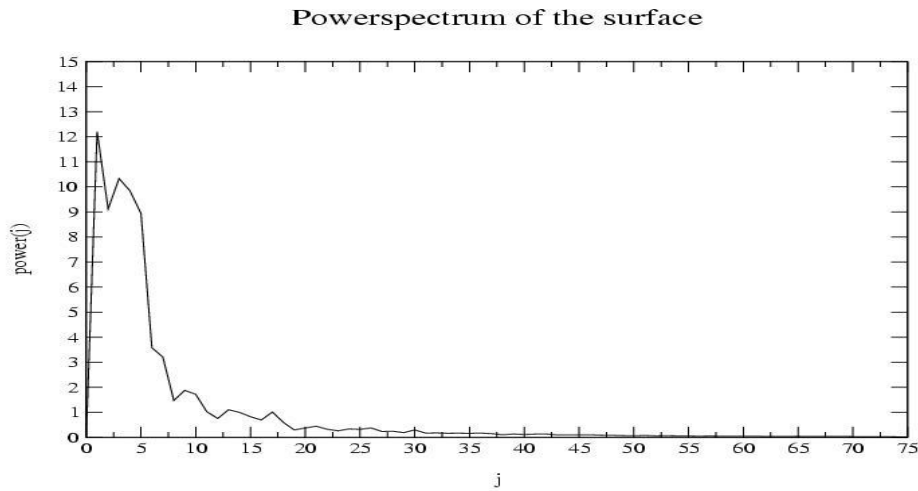
3.2.2 Using hanalysis to produce power spectra

Power spectrum of the simple harmonics

Power spectrum of low degree surface harmonics
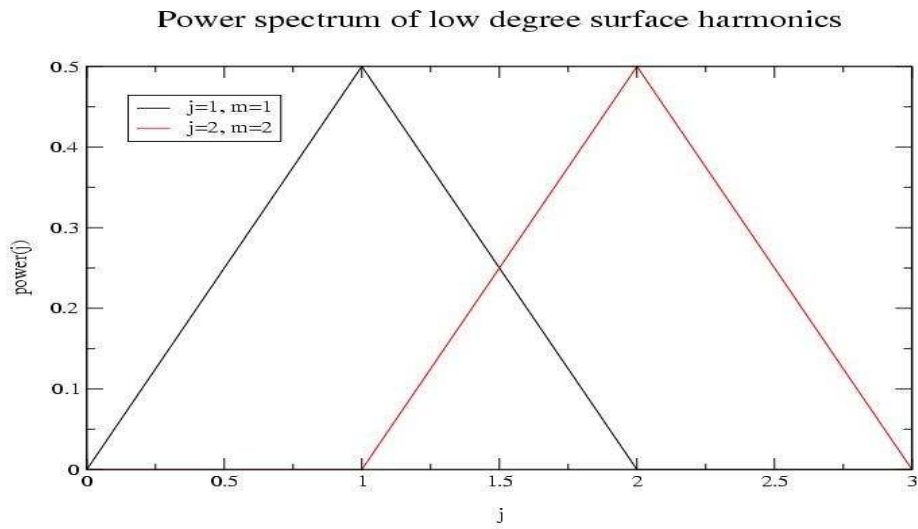
This graph shows that for j=1 and m=1 the max power(j) value is at 0.5 and logically for j=2 and m=2, the same but this time at j=2.

Power spectrum of the surface topography

Powerspectrum of the surface

This graph was generated using jmax=99. It shows that the degrees higher than 20 are not very relevant for higher precision in further calculations. So from then on we used jmax=20.
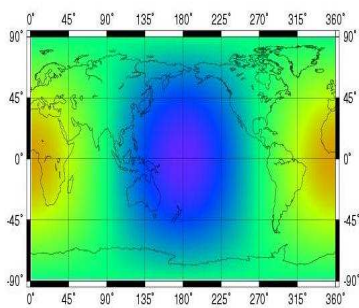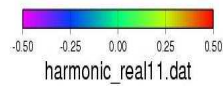
Power spectrum of the crustal thickness
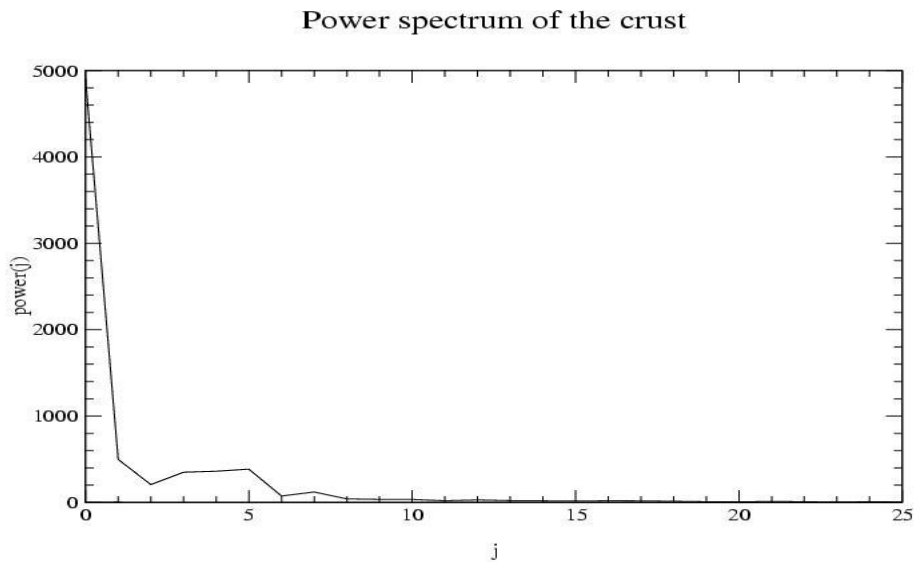
This graph shows that after jmax=10 the higher values for j do not give more precision and are therefore not very relevant. Both for comparison purposes with the surface topography, we still used jmax=20.

3.2.3 Application of hsynthesis to check whether the results compare from 3.2.1

Harmonic real 1*1 GMT plot   Harmonic 2*2 GMT plot
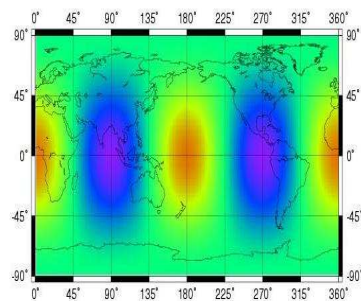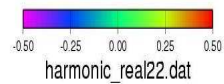
Both GMT plots compare with the low degree surface harmonics GMT plots in 3.2.1. Only the ranges in both plots slightly differ, but the overall picture is the same. The code has therefore successfully been tested.

3.2.4 Writing a geoidcoef programme to compute the spherical harmonic coefficients of the effect on the geoid of surface topography.

```
      program geoidcoef
      implicit none

c     declaration of variables
      double precision grav, rearth, rhozero, pi, g, deltarho
      integer delta, phi, nmax, n, maxdegree, r, i
      integer index,j,jdummy,mdummy,m
      real JMAX,MAXHJM, MAXDELTAN
      parameter (JMAX=90,MAXHJM=JMAX*(JMAX+1)/2+JMAX+1)
      parameter (MAXDELTAN=JMAX*(JMAX+1)/2+JMAX+1)
      complex HJM(MAXHJM)
      complex DELTAN(MAXDELTAN)

c     open input and output file
      open(UNIT=9, FILE='surf.spc')
      open(UNIT=10, FILE='surftopometer.spc')
      read(9,*) maxdegree

c     Constants and starting values in metres (SI units)
      pi = 3.141592
      nmax = 0
      grav = 6.67732E-11
      rearth = 6371E+3
      rhozero = 2700
      r = 6371E+3
      g = 9.81

      write(10,*) maxdegree, '# MAX. HARMONIC DEGREE'

      do j=0,maxdegree
        do m=0,j
          index = j*(j+1)/2+m+1
          read(9,*) jdummy, mdummy, hjm(index)
c     Formula for calculating deltaN, the geoid effect
          deltaN(index) = (((4*pi*grav*rearth)/((2*jdummy) + 1))
     1     *rhozero*hjm(index))/g
          write(10,*) j, m, deltaN(index)
        enddo
      enddo

      end
```
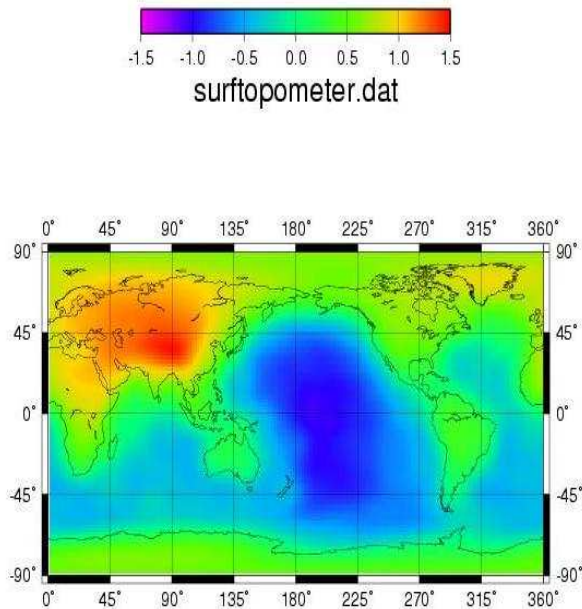
GMT plot of the geoid (surface topography only)

surftopometer.dat

This plot shows that the continental crust has a positive effect for the geoid, whereas for oceanic crust the geoid shows a negative effect. The greatest positive effect is at the himalayas and Eurasia as a whole. The greatest negative effect is in the pacific ocean. At the oceanic-continental boundaries the effect is almost zero.

3.2.5 Evaluation of deltagr(r,theta,phi) on the same mesh for different positions above the Earth's surface, r-Rsurf = 1, 10, 100, 1000 km.

```
      program gravity
      implicit none

c     declaration of variables
      double precision grav, rearth, rho, pi, variable
      integer delta, phi, nmax, n, maxdegree, i
      integer index, j, jdummy, mdummy, m
      double precision ca, JMAX, MAXHJM, MAXDGRAV, MAXCB
      double precision cc, cd, ce, r
      parameter (JMAX=90,MAXHJM=JMAX*(JMAX+1)/2+JMAX+1)
      parameter (MAXDGRAV=JMAX*(JMAX+1)/2+JMAX+1)
      parameter (MAXCB=JMAX*(JMAX+1)/2+JMAX+1)

      complex CB(MAXCB)
      complex HJM(MAXHJM)
      complex DGRAV(MAXDGRAV)

c     open and create input and output file
      open(UNIT=9, FILE='surf.spc')
      open(UNIT=10, FILE='grav10.spc')

      read(9,*) maxdegree

      write(6,*) 'give height above earth surface in kilometres
      read(5,*) variable
```

```
c    Constants and starting values
     pi = 3.141592
     nmax = 0
     grav = 6.67732E-11
     rearth = 6371
     rho = 2700
c    Radius of the earth + distance from the earth's surface
     r = 6371 + variable
     ca= 4*pi*grav*rearth*rho

     write(10,*) maxdegree, '# MAX. HARMONIC DEGREE'

     do j=0,maxdegree
      do m=0,j
        index = j*(j+1)/2+m+1
        read(9,*) jdummy, mdummy, hjm(index)
c    Parts of the formula simplified as ,ca, cb, cc, cd, ce
     cb(index) = ca*(hjm(index))
     cc = dble((2*jdummy)+1)
     cd = rearth**(dble(jdummy+1))
     ce = dble(jdummy+1)
c    Formula for calculating the change in gravity (dgrav) above earth's surface (ddeltaU/dr)
        dgrav(index) = ((cb(index))/cc)*cd*ce*(1/r)**ce
        write(6,*) jdummy, cb(index), cd
        write(10,*) j, m, dgrav(index)
      enddo
     enddo

     end
```
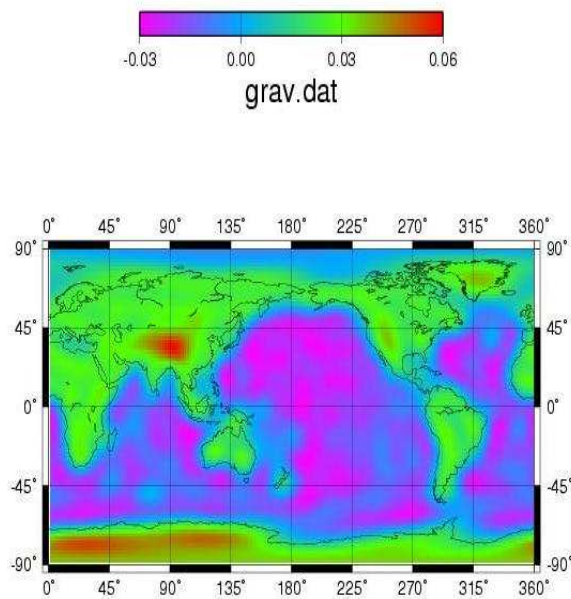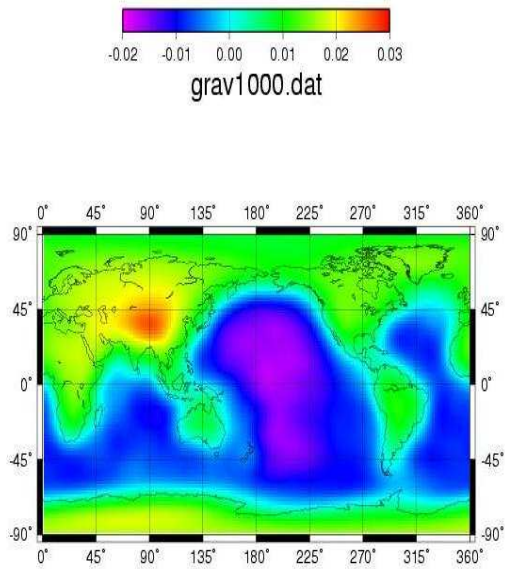
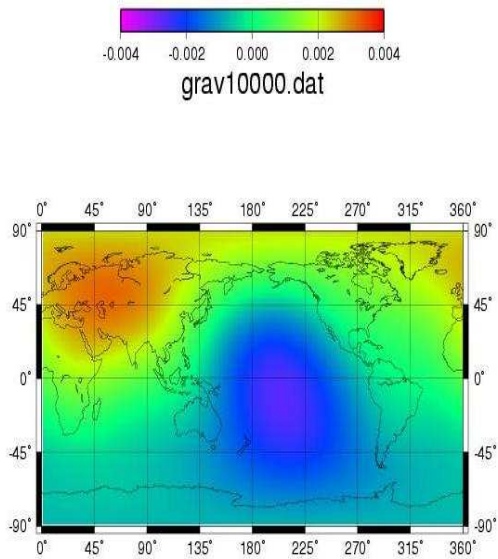GMT plot of the change in gravity 1, 10, 100 km above earth's surface

There is a slight decrease in the change in gravity between 1, 10, and 100 km above the earth's surface, but it is not discernible in the GMT plots, hence only one is shown here.

GMT plot of the change in gravity 1000 km above earth's surface

GMT plot of the change in gravity 10000 km above earth's surface

Summary

Above the continents the change in gravity is positive and above the oceans the change is negative. As can be seen from the plots for 1000km and 10000km above the earth's surface, the change in gravity continues to get smaller. Furthermore, the fine details and sharp boundaries become vague and diluted. The differences in the results for different observational heights can be explained by looking at the derived formula. You divide by r, and as r increases significantly, dgrav (change in gravity) becomes increasingly smaller. The dilution of the boundaries is due to a decrease in resolution the further away from the earth.